



WHITE PAPER

# Fading Basics



Narrowband, Wideband, and Spatial Channels

## Introduction

Radio technologies have undergone increasingly rapid evolutionary changes in the recent past. The first cellular phones used narrow-band FM modulation, which was soon replaced by digital modulation in second and third generation devices. Today, multiple-antenna systems are being employed to increase data rates. These provide improved quality while decreasing operational costs.

As technology progresses to take advantage of more complex channel characteristics, the channel modeling required to emulate the radio environment for testing becomes both more critical and more complex. For instance, when bandwidths are increased (to support higher data rates) receivers become more susceptible to Inter-Symbol Interference (ISI). To ensure that measurements in the lab accurately correlate to the quality of the user's experience, channel models must account for all aspects of the radio environment.

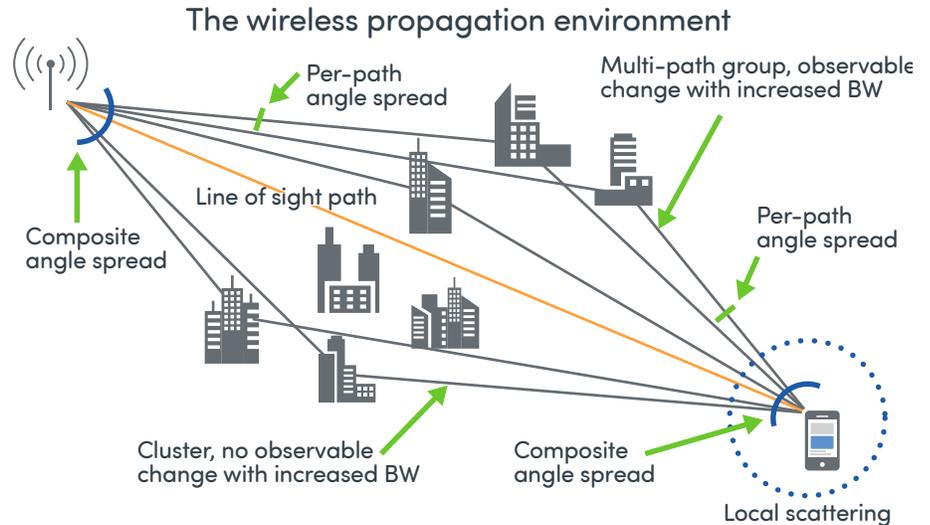


Figure 1: The wireless propagation environment.

Figure 1 illustrates the modern radio propagation environment, consisting of a number of different components. Referring to the figure, a signal transmitted from the base station to a subscriber is shown to consist of a number of paths. This is generally referred to as multi-path propagation. For purposes of system design and testing, a limited number of paths, usually 4-24, are used to model the radio channel.

The radio bandwidth determines the number of paths required to produce an adequate model. Each path is made up of a number of sub-paths, representing individual plane waves received from nearby reflections. Multiple sub-paths are closely associated with a single path and may not be observable in the received signal. In reality, each sub-path arrives at the receiver with a slightly different time delay and Angle of Arrival (AoA). These sub-paths characteristics cause each path to have its own characteristic Delay Spread (DS) and Angle Spread (AS). Due to bandwidth limitations, the Path DS is usually considered to be zero. This means it is assumed that all the sub-paths arrive at the same time.

For narrowband channels (such as early FM radios operating at 25-30kHz bandwidth), a receiver cannot resolve the different paths. In this case, the receiver sees a single composite signal which is the vector sum of all the multi-path components.

This is illustrated in Figure 2, which further describes the effects of bandwidth on a receiver's ability to resolve multi-path components. Wideband measurements indicate that each individual path tends to be received from a particular direction and with a limited AS. Figure 2 is a conceptual description of a signal received on a wideband radio. The powers of the five multi-path components are shown with their peak normalized to zero dB. The paths are represented by the colors: red, blue, black, magenta, and green, which are used to denote power received at each of the delays. Each path has a unique power-angular distribution or Power Azimuth Spectrum (PAS) at its given delay.

The five paths shown are made up of many distinct sub-path plane-waves, received at slightly different AoAs. It is assumed that all sub-paths for a given path are received as a cluster and arrive at the same time.

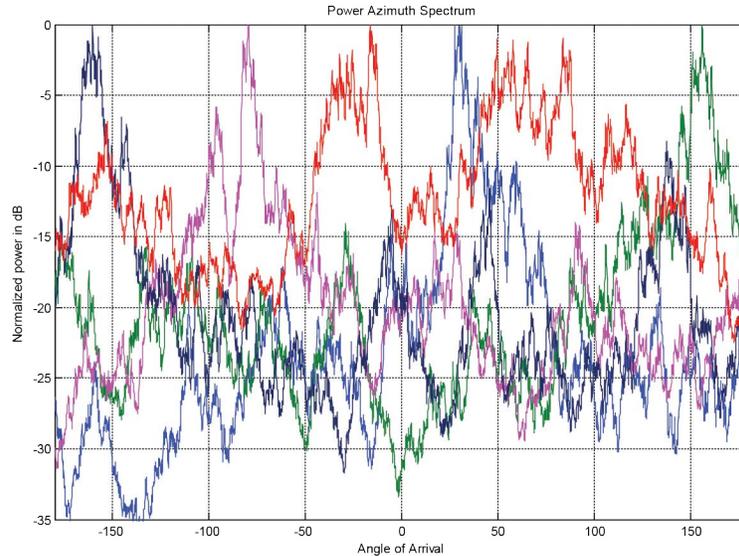


Figure 2: Conceptual Power Azimuth Spectrum for a wide bandwidth signal.

When the signal in Figure 2 is received by a narrowband receiver, all multi-path components are indistinguishable and are combined together at the antenna. The Power Azimuth Spectrum is nearly uniform for this case, and results in classical Rayleigh Fading.

In general, as the bandwidth increases, so does the ability of the receiver to resolve the different paths, thus increasing the number of paths required by an accurate channel model. As the number of observable paths increase, the statistical characteristics of fading change; while the narrowband model treats multiple paths as a single composite path, the wideband model requires multiple paths.

Referring back to Figure 1, increased bandwidth (BW) distinguishes a multi-path “group” from a cluster. A cluster is defined to be the source of a reflected path that can not be separated into additional paths by increasing the bandwidth. A multi-path group, however, will become resolvable into separate paths with increased bandwidth. In the limit case with infinite bandwidth, every “path component”, no matter how insignificant, is resolvable, producing thousands of paths. However, actual bandwidths filter our ability to resolve different paths and lead to lower and more practical numbers of paths we use in today’s models.

Each path is shown to depart the antenna with an angle spread (AS) expressed in degrees. In reality, signals leave even highly-directional antennas in all directions, but only certain paths reach the mobile station (MS) with receivable levels of power. Only these paths are used in modeling the channel.

When the transmit frequency is the same at the base station (BS) and MS (for example, in Time Division Duplex [TDD] systems), the path is identical in either direction. This bi-directional equivalence at a given frequency is called reciprocity. This principal can be used to understand how paths behave, and why only certain paths are modeled. In Frequency Division Duplex (FDD) systems, different frequencies are used in each direction. Since the frequency is not the same, the paths are not reciprocal, but their average powers are still highly correlated.

The AS of an individual path is different at the BS and the MS due to scattering near the antenna. Since the subscriber is near the ground and in the presence of more clutter, there are reflections near the MS antenna, which leads to a larger AS than what is observed at the BS. Figure 3 presents a top view of a subscriber in an urban area, where a path is arriving at the subscriber antenna. The individual “rays” depicted in the figure are sub-paths, and are the components that make up a path.

Because the small differences in the length of travel for these sub-paths are non-resolvable in typical channel bandwidths, these sub-paths act together as a path. The sub-paths are combined at the receive antenna and produce a faded signal due to the vector sum of sinusoids of varying phases.

The arrival of sub-paths from a variety of directions causes the path to have an AS. Sub-paths that appear from “behind” the MS are generally weak since a reflection with a high angle of incidence produces a weaker signal than those with low angles of incidence. This produces the tendency towards narrow angle spreads (as is observed in measured data). The strongest receive paths tend to be the most narrow, because they receive a dominant signal, and reflections contribute less to the result.

When all paths are combined, a “composite” angle spread can be calculated, (not to be confused with the path AS described above.) The composite angle spread is different at the base station and at the subscriber’s location, due to the unique propagation effects present at each end of the radio link.

The average angle associated with the angle spread represents the Angle of Departure (AoD) or Angle of Arrival (AoA) of the signal. When a path is completely specified at each end, the channel can be described as a “double directional” channel.

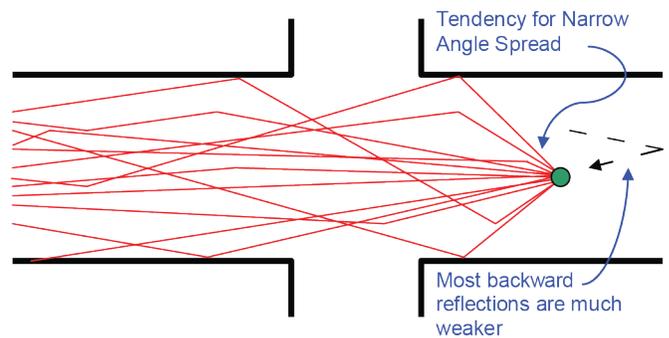


Figure 3: Angle spread at the subscriber.

# Fading Models

## Flat Fading

Since multiple paths are not resolvable in narrow bandwidths, the RF environments seen in narrowband FM technologies such as AMPS, NAMPS, TACS, can be modeled by “flat” fading channels. These frequency-flat channels fade the same amount across the frequency band and are easily modeled by single-path fader models such as the Jakes Fader, JTC Fader, or others.

The flat fading signal illustrated in Figure 4 represents a signal path faded due to reflections produced by localized clutter. Technically, this signal and the fading associated with it are due to multi-path reception. However, the term multi-path is more commonly applied to delayed paths leading to delay spread, and this single-path fading behavior is the result of local scattering.

In this example, a 30m radius of scattering is used, producing many reflections of the signal in close proximity to the subscriber. These reflected signals are equivalent to the sub-paths described earlier and are received at various levels and phases and combined at the subscriber’s antenna. Because the bandwidth of 30kHz is so small, there is virtually no difference between the fades across the band. The flat fading characteristic remains true even with the addition of multiple delayed paths, even with fairly lengthy delays (tens of  $\mu$ s), since these are not resolvable in narrowband channels.

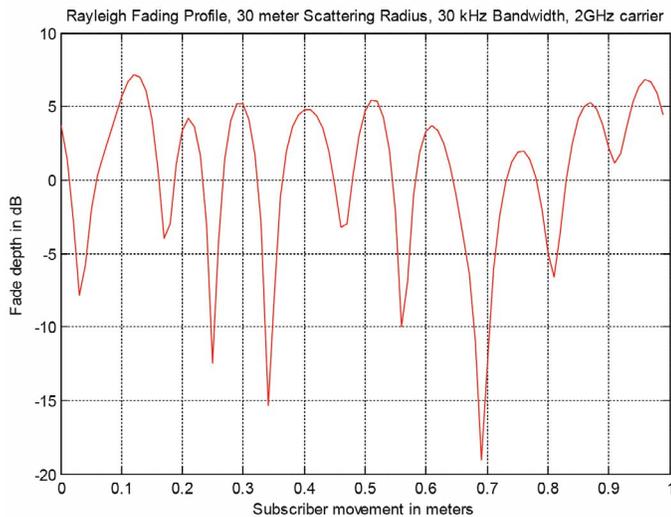


Figure 4: Sample Rayleigh-Faded narrowband 30kHz signal.

## Frequency Selective Fading

Digital radio technologies, including CDMA, WCDMA, UMB, LTE, and WiMAX, transmit digital signals in a bandwidth larger than the coherence bandwidth of the channel. This means that the channel no longer looks “flat” across the frequency band; rather, the fading is “frequency-selective” with different signal strengths present at different frequencies across the band. Figure 5 illustrates a single-path channel with the same 30m scattering radius as shown in Figure 4. This channel represents a single strong path that is faded due to localized clutter, with many reflections in close proximity to the subscriber. Note from the Figure that the frequency is stepped across the 5MHz bandwidth in 0.5MHz steps (shown by various color traces) and illustrates how the fading changes with frequency.

Typically, more than one strong path is received, each having a delay based on the distance the signal travels. This is called multi-path propagation. As each delayed path arrives at the receiver, it is scattered by local clutter. When multiple paths are added together at the receiver, each with progressively longer delays; the combined signal exhibits the same frequency-selective fading behavior as the locally scattered path, but the change with frequency is much more rapid. The channel looks coherent (the same) over a much smaller bandwidth when multi-path is present, and has a lower coherence bandwidth than the single-path case.

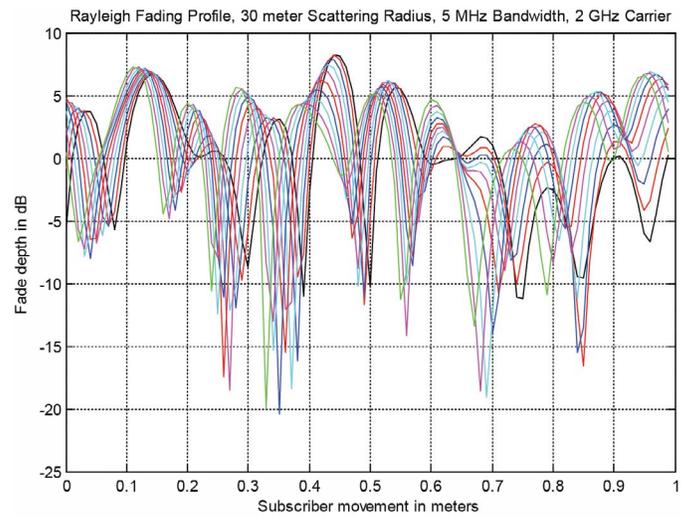


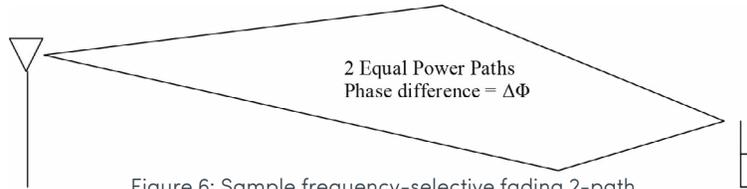
Figure 5: Sample Rayleigh-Faded 5MHz wide signal.

## Why is a Signal Frequency-Selective?

Consider an example of two paths (phasors) having equal power but different path lengths  $d_1$  &  $d_2$ , as shown in Figure 6. Each signal path has a phase measured at the receiver equal to:

$$\Phi_1 = \frac{d_1 \cdot 2\pi}{\lambda}$$

$$\Phi_2 = \frac{d_2 \cdot 2\pi}{\lambda}$$



The relative phase difference  $\Delta\Phi = \Phi_2 - \Phi_1$  is a function of frequency and the differential path length  $d_2 - d_1$ .

$$\Delta\Phi = \frac{2\pi(d_2 - d_1)}{\lambda}$$

At the given receiver location,  $\Delta\Phi$  increases in phase as the frequency is increased. Every time  $\Delta\Phi$  rotates by  $2\pi$ , there is a phasor addition and a phasor cancellation of the two path summation, i.e., at 0 and  $\pi$ . This interaction between the two paths produces frequency selective fading.

This can be seen by the phase relationship:

$$\Delta\Phi = \frac{2\pi(d_2 - d_1)}{\lambda} = \pi \cdot (2n - 1)$$

where  $n$  is an integer number representing phase differences that are odd multiples of  $\pi$ , which represent the frequency selective fades.

$$\text{Thus: } \lambda = \frac{2(d_2 - d_1)}{2n - 1} = \frac{2\Delta d}{2n - 1}$$

$$\text{Or: } f = \frac{(2n - 1)C}{2(\Delta d)}$$

This formula means that every time the frequency is an integer multiple of  $C/(2\Delta d)$ , there is a frequency-induced fade in the receiver.

Consider the frequency over a bandwidth:  $f_2 - f_1 = \Delta f$  where  $n_2 = n_1 + m$ , and  $m$  is an integer representing  $m$  fades across the band.

$$f_1 = \frac{C(2n_1 - 1)}{2(\Delta d)}$$

$$f_2 = \frac{C(2n_2 - 1)}{2(\Delta d)}$$

Now let  $n_2 = n_1 + m$ , where  $m$  is an integer representing  $m$  fades across the band.

$$f_2 - f_1 = \Delta f = \frac{C(2(n_1 + m) - 1)}{2(\Delta d)} - \frac{C(2n_1 - 1)}{2(\Delta d)} = \frac{mC}{\Delta d}$$

- Note that when  $m$  is a non-integer, there may be a counting ambiguity where depending on the starting phases (which is a function of the path difference) the BW may contain  $m$  or  $m+1$  fades. Since the equation was derived assuming  $m$  is an integer then we should only obtain  $m$  fades, so this should not be a problem.

Therefore,

$$\Delta f \Delta d = mc$$

Where  $\Delta f$  = the bandwidth in Hz

$\Delta d$  = the path length difference in meters

$m$  = an integer number representing the number of fades across the band

$C$  = speed of light

Consider the path differences required to observe a fade in a given receiver bandwidth; for example, achieving an odd multiple of  $\pi$  phase shift between the two paths.

Receiver Bandwidth $\Delta f$	Path Difference for one fade across the band $\Delta d = \frac{mC}{\Delta f}$	# of fades for a 60 m path difference $m = \frac{\Delta f \Delta d}{C}$	# of fades for a 600 m path difference $m = \frac{\Delta f \Delta d}{C}$
30kHz	10 km	0.006	0.06
20MHz	15 m	4	40

Table 1: Frequency selective fading sample for a 2 path model.

Figure 7 illustrates two of the examples shown in Table 1, where a path delta of 15 and 60 meters are shown to experience 1 and 4 fades respectively in a 20MHz BW.

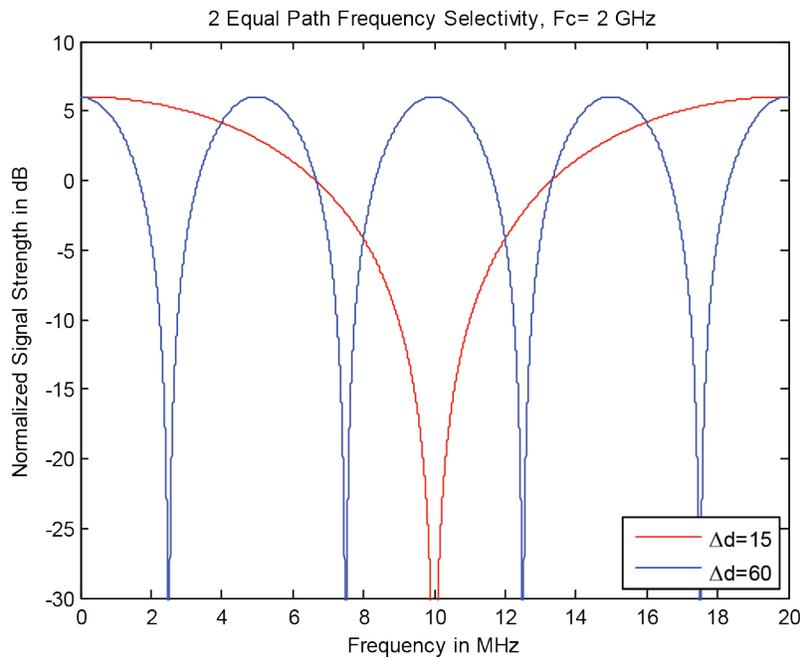


Figure 7: Frequency selective fading for some path length differences.

Notice that narrowband radio will probably not have significant problems with frequency-selective fading. This is because it would require path differences of at least 10km with roughly equal path power levels (since a null requires cancellation). For typical path differences, only a small fraction of a fade can be seen across the band in the narrowband case.

For 20MHz wideband radios, frequency selective fading is quite evident. For typical path delays, there may be 20-40 or more fades across the band.

This concept is also illustrated in Figure 8 and Figure 9, where a path difference is shown at two separate measurement frequencies. Because the wavelength changes with frequency, so does the number of sine-wave repetitions within the fixed path difference. The phase difference of the two signals (resulting from the path difference), is dependent on frequency, and each sine-wave repetition that occurs within the path difference represents a frequency selective fade in the band.

Now at Frequency 1 in Figure 8, the phases between the two paths nearly align, so the signals are adding constructively. At Frequency 2 in Figure 9, the phase difference has increased and the signals from the two paths are out of phase and are cancelling. Therefore, the fading produced by a two-path signal is frequency selective.

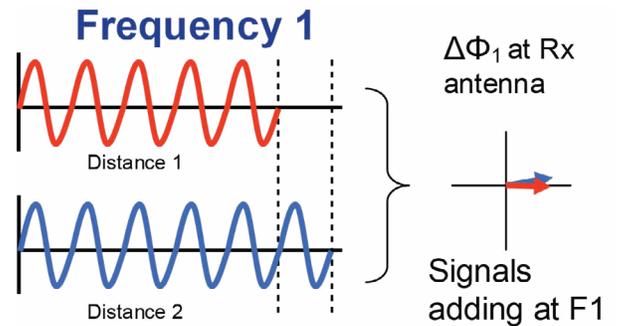


Figure 8: Two path sample at frequency 1.

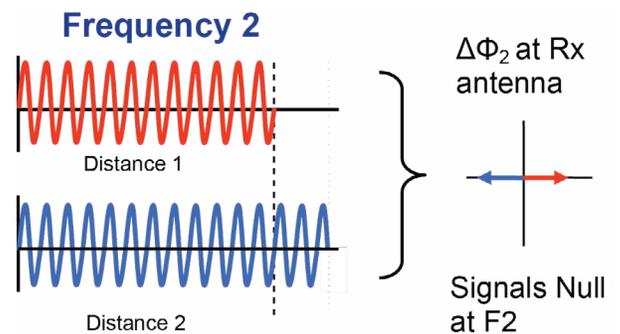


Figure 9: Two path sample at frequency 2.

### Multi-Path Delays and Delay Spread

Frequency-selective channels result from multi-path propagation. As shown in Figure 5, local scattering can exhibit frequency selectivity, but it is more evident from the combination of distinct paths. Since the delays between paths are much larger than the delays within a path (i.e., the small intra-path delays due to local scattering,) these latter delays are typically ignored and assumed to have zero delay spread. Therefore each path will experience flat fading. The frequency selectivity of the channel is then only a function of the relative path delays.

These delayed paths are illustrated in Figure 10, again assuming no intra-path delay. Each path has a unique delay time and relative power, as shown.

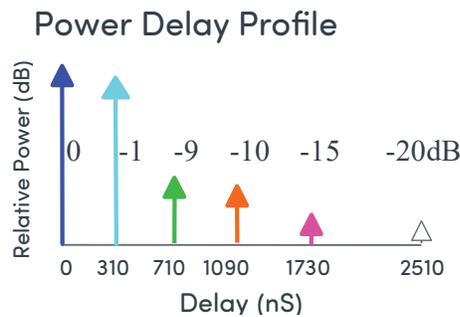


Figure 10: Sample power delay profile from the ITU Vehicular A Channel Model.

### Multiple Antennas

When multiple antennas are used at the transmitter, the receiver, or both, significant improvements in performance can be obtained. Multiple Antenna techniques vary and include simple diversity selection and combining schemes, and more complex approaches like beamforming, and Multiple-Input-Multiple-Output (MIMO) systems.

Modeling multiple antenna approaches requires a fading channel with the proper correlation between antenna branches. High correlation means that the signals are very similar, so both branches may experience a strong or weak signal at the same time, making it harder to withstand a given fade. Low correlation means that the signals are more random, such that a fade on one branch might be mitigated by a stronger signal on the other branch.

Early models set the correlation between antenna branches to an average value for evaluating simple diversity receivers. Today, more complex models are required since new air interfaces are designed to adapt between different techniques based on the dynamics of the channel. These may include a variety of multiple antenna techniques including Beam Forming, MIMO, Space Division Multiple Access (SDMA), and scheduling approaches like frequency-selective scheduling. Multiple antennas are usually expressed as an MxN combination, where M is the number of antennas at the transmitter, and N is the number of antennas at the receiver. Typical configurations may include 1x2, 2x2, 4x1, 4x2, 4x4, 1x4, and others.

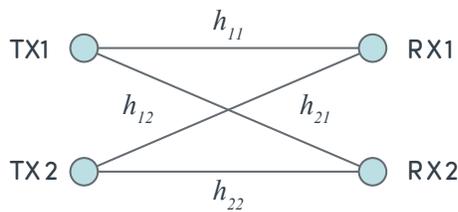


Figure 11: 2x2 Multiple antenna configuration.

A 2x2 example is shown in Figure 11, where a total of 4 connections are present between antenna elements. These connections are indicated by the  $h_{11}$ ,  $h_{21}$ ,  $h_{12}$ , and  $h_{22}$ , each representing a connection between the base and the subscriber. Each connection has a complex path gain (for example, amplitude and phase) measured with respect to a normalized average power. These terms are grouped together to form an  $H$  matrix as shown in Figure 12. There is a unique  $H$  matrix for each delayed path. For example, a six-path channel will have six  $H$  matrices that will be updated quickly enough to track the Rayleigh fading of each path.

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

Figure 12: Complex channel matrix  $H$ .

The signals at the transmitter and receiver antenna elements are correlated, not random. Extensive measurements have shown that the correlation is not constant, but varies significantly over a geographic area or drive route. The correlation between antenna elements is a mathematical function related to the make-up of the local scattering and is a function of the signal AS, its AoA, and the subscriber's direction of travel (DoT).

The Power Azimuth Spectrum (PAS) of each path is typically modeled by a Laplacian distribution wherein the signal drops off exponentially (linearly in dB) as the angle increases in magnitude from the average direction of arrival.

Figure 13 illustrates the complex correlation that results from the Laplacian PAS when a 2° AS is specified for BS antennas separated by four wavelengths. The magnitude indicates that the correlation between antenna elements is quite high, ranging from about 0.7 to 1.0 (where a value of 0 represents no correlation and a value of 1.0 represents perfect correlation).

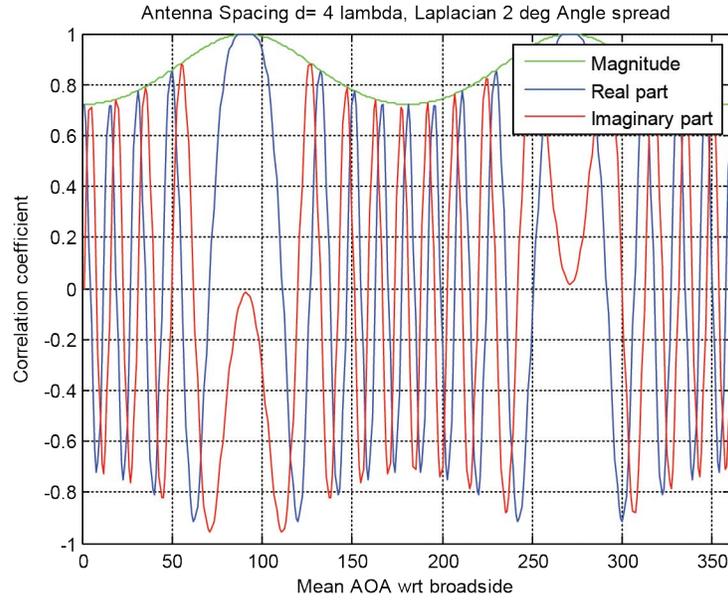


Figure 13: Base station antenna correlation.

Figure 14 illustrates the correlation between subscriber antennas separated by half a wavelength. While the antennas are closer together, the correlation is somewhat lower due to the larger AS ( $35^\circ$ ). Using such an array at the subscriber is assumed for simplicity. Multiple antenna configurations may actually include polarized antennas to obtain low correlation with antenna elements in close proximity. For both the base station and subscriber, the antenna correlation is a function of the path angle.

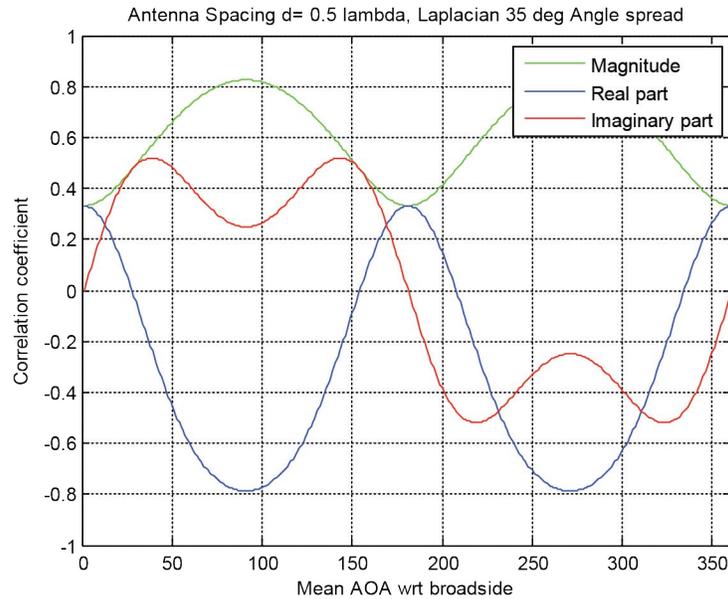


Figure 14: Subscriber antenna correlation.

- It should be noted that averaging the complex correlation across angles of arrival  $0-2\pi$  will result in exactly the correlation of the uniform  $\rho = -0.304$ .

Correlation can be plotted in another way, as shown in Figure 15. This is based on the separation of antenna elements or the distance between samples. These two interpretations are exactly the same, and indicate how much the signal is changing versus distance.

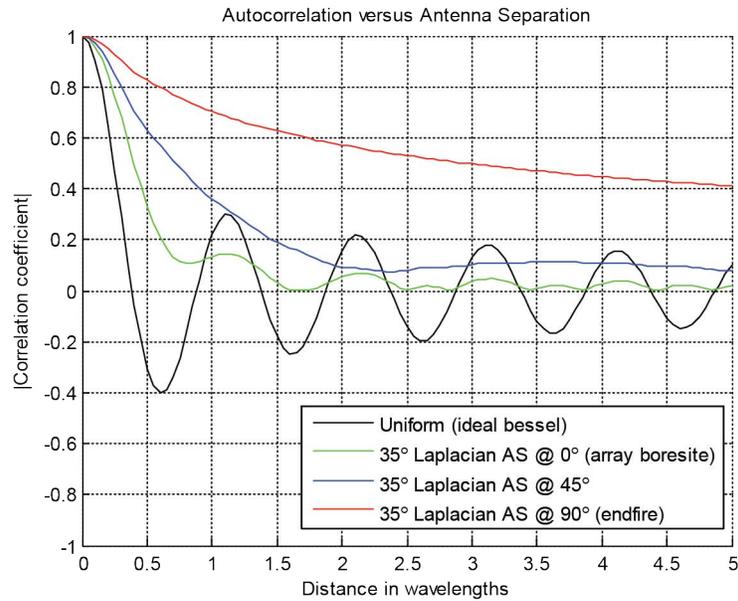


Figure 15: Autocorrelation.

Note that using the narrow angle spread, selected to match field measurements, increases correlation as compared to the uniform or classical Doppler assumption. This result causes a reduced fading rate which changes as a function of AoA.

## Wideband Channels

As described earlier, when channel bandwidth increases so does the ability to resolve multi-path. In the narrowband case, it is typically assumed that all path sub-components arrive at the same time, i.e., the path delay spread is zero, leading to frequency-selective fading by the interaction between paths. However, for extended bandwidths ( $\geq 20\text{MHz}$ ) it is desirable to have either more paths, or some intra-path delay spread to enhance the modeling of frequency selectivity.

Most channel models to date have been limited to a small number of paths since channels were only a few MHz wide. With wider bandwidths, the Spaced-Frequency Correlation Function [ ] exhibits periodic oscillation across frequency and describes how different frequencies are correlated across the band. Figure 16 shows the result for the Vehicular A channel model, described earlier in Figure 10. The oscillations in correlation are due to the limited number of paths, wherein the differences in path lengths contribute a different amount of phase at each frequency. As the frequency is increased, the phases advance and produce the oscillation. When the complex path components are combined, there are some frequencies in which paths cancel and other frequencies in which paths add constructively. When fading is added to these paths, the fading is correlated across frequency based on the phase relationships between paths.

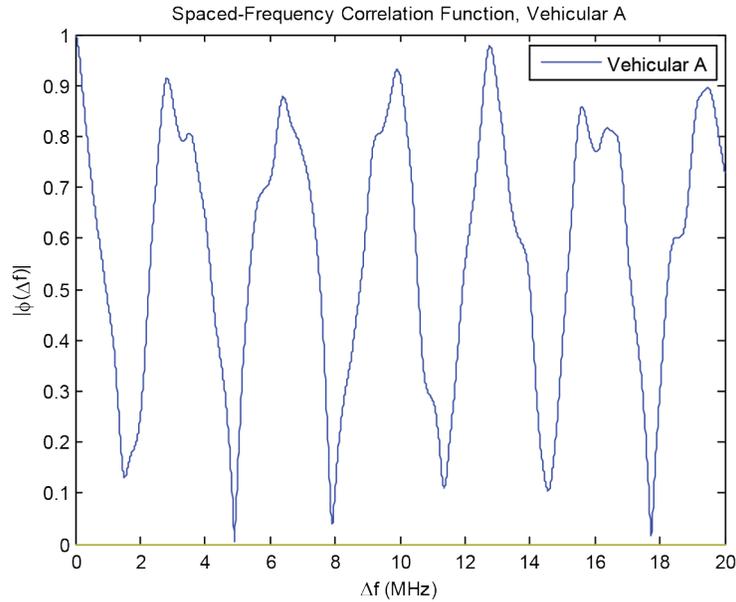


Figure 16: Spaced-frequency correlation function.

A Spaced-Frequency correlation function exhibiting this oscillatory behavior, shown in Figure 16, is sometimes undesirable, such as when modeling frequency-selective schedulers. Wideband measurements indicate that the spaced-frequency correlation of actual channels drops to a low level and remains low. To reduce the level of oscillation and improve the wideband characteristics of the channel, various improvements can be made by adding additional paths or by splitting one or more existing paths into multiple delayed paths, sometimes called mid-paths.

## Shadowing

Shadow fading (SF), also called slow fading or log-normal shadowing is the variation in average received power from one location to another. This log-normally distributed parameter is generally independent of path loss; for example, the distance from the BS, and represents the variation due to shadowing or “blockage” from clutter on the ground.

SF is typically correlated in two ways. First, it is correlated with distance, where the shadow fading value changes slowly with movement of the subscriber. This distance, referred to as a de-correlation distance, is typically tens of meters in urban areas and a few hundred meters in suburban and rural areas. This distance is descriptive of the size of the clutter that obstructs the path to the subscriber. Individual buildings may be the main component of the clutter in urban areas, whereas city blocks or terrain changes may be the clutter in suburban areas.

Shadow fading can also be correlated with respect to the angle of the paths between different BS sites. There will be a common component of the shadowing present at the subscriber’s location, along with a difference component of the shadowing for the path to each base station. This difference component is present since each path is unique and sees clutter that is unique. The common component of the shadow fading produces a correlation and this is called site-to-site correlation. This is often modeled with a constant 50% correlation.

## Channel Modeling and MIMO Capacity

It is important to have channel models that correctly emulate real-world conditions in order to adequately simulate multiple antenna performance. Since algorithms will be compared and optimized based on channel models, the models must include proper correlation between antenna elements. Adding correlation diminishes the ability of a MIMO transceiver to spatially separate the channel into orthogonal components in order to support additional transmission streams. Thus, high degrees of correlation limit the potential capacity of MIMO, and therefore must be included in a Spatial Channel model.

The narrow angle spreads described in Figure 14 and Figure 15 were selected to account for this increased correlation between antenna elements.

The effect of the correlation is apparent in how the MIMO capacity is described in the following equation[iii]. This equation gives the instantaneous capacity value, as shown in Figure 17.

$$C = \log_2 \left( \det \left( \mathbf{I} + \left( \frac{\Phi}{m} \right) \mathbf{H} \mathbf{H}^H \right) \right) \text{ bps/Hz Instantaneous Capacity}$$

Where:

$\mathbf{H}$  is the channel matrix of complex path gains,

$\mathbf{I}$  is the identity matrix,

$\Phi$  is the average SNR,

$m$  is the number of transmit antennas, and

$\mathbf{H}^H$  is the complex conjugate transpose.

This equation is quite interesting. It is very similar to the Shannon capacity formula but the  $(1+\text{SNR})$  is replaced with a matrix equation.

The determinant of a matrix is always the product of eigenvalues of the matrix:

$$\det(\mathbf{H}\mathbf{H}^H) = \lambda_1 \lambda_2 \cdots \lambda_N$$

Adding an identity matrix shifts the eigenvalues of a matrix:

$$\det(\mathbf{I} + \mathbf{H}\mathbf{H}^H) = (1 + \lambda_1) (1 + \lambda_2) \cdots (1 + \lambda_N).$$

Taking the log of a product is equivalent to summing the individual capacities on each eigenvalue.

$$C = \sum \log_2(1 + \alpha_i \text{SNR}), \text{ where: } \alpha_i = \lambda_i \frac{\Phi}{m}$$

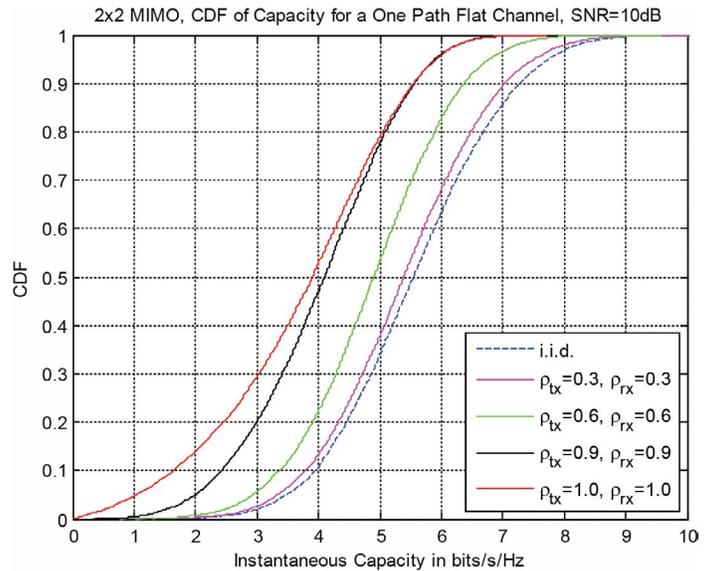


Figure 17: Instantaneous MIMO capacity.

The average capacity for the MIMO system is given by:

$$C = E \left\{ \log_2 \left( \det \left( \mathbf{I} + \left( \frac{\Phi}{m} \right) \mathbf{H} \mathbf{H}^H \right) \right) \right\} \text{ bps/Hz Average Capacity}$$

Where:

$E$  is the expected value over the random channel  $\mathbf{H}$ .

When correlation is added to the path gains, the terms in  $\mathbf{H}$  become less random. This results in a diminished ability to spatially separate the channel into its constituent eigenvalues, which are the orthogonal components of the channel capable of supporting a transmission. Therefore, the highest capacity would be possible when each element of the  $\mathbf{H}$  matrix is i.i.d. Rayleigh faded signal representing no correlation between elements. Once the correlation between antenna elements is included, the capacity is reduced. If the Spatial Channel Model (SCM) is used instead of ideal (i.e., Rayleigh) fading, the correlation can be much higher, significantly reducing the ideal capacity of a MIMO system.

## About Spirent

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Because the correlation predicted by the SCM is a function of antenna orientation, AoA, AS, DoT, etc.; it is important to analyze the capacity in terms of a system simulation. This is shown in Figure 18 below.

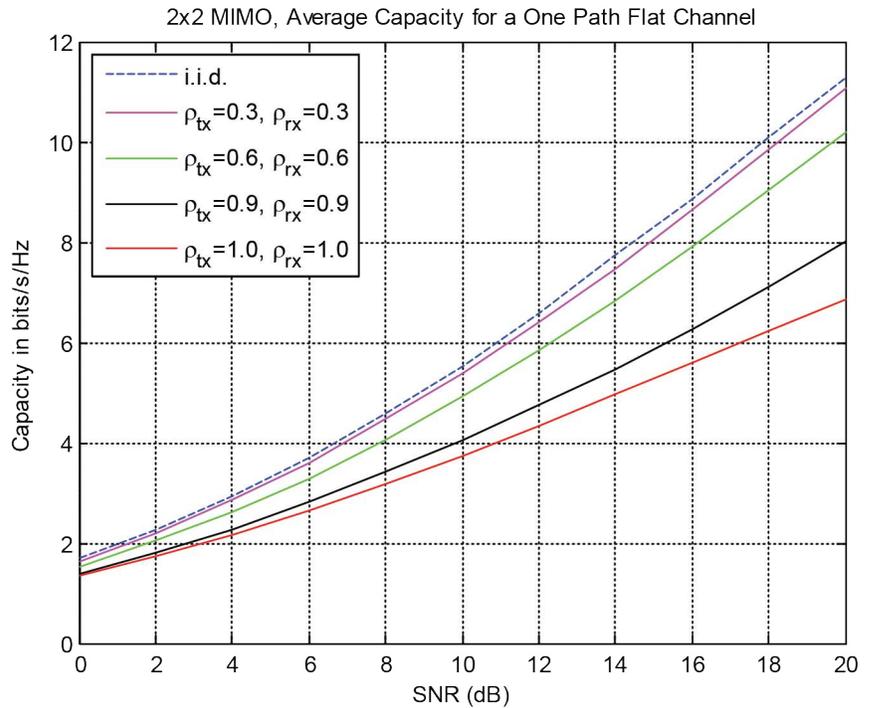


Figure 18: Average MIMO capacity.

3. COST-259 Final Report, Wireless Flexible Personalised Communications, COST 259: European Co-operation in Mobile Radio Research, Edited by Luis M. Correia John G. Proakis, Digital Communications, 3rd Edition, McGraw-Hill, 1995.

G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Personal Communications., vol. 6, no. 3, pp. 311-335, Mar. 1998.